Q.P. Code : 15123

First Semester B.C.A. Degree Examination, November/December 2019

(CBCS – (Freshers & Repeaters)

Computer Science

Paper 105 T – DISCRETE MATHEMATICS

Time : 3 Hours] *[Max. Marks : 100*

Instructions to Candidates : Answer all Sections.

SECTION – A

I. Answer any **TEN** of the following. Each question carries **2** marks : **(10 × 2 = 20)**

1. If $A = \{2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3\}$ find $A \cap B$.

2. If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{0, 2, 3\}$, find $(A \cap B) \times C$.

3. Define Tautology.

4. Define Scalar matrix with example.

5. If $A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$, find $2A + 3B$.

6. Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

7. If $\log_7^x + \log x_7^2 + \log x_7^3 = 6$, find x.

8. Define permutation and combination.

9. Define an abelian group.

10. If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\vec{b} = 3\vec{i} - 4\vec{j} - 5\hat{k}$, find $|\vec{a} + \vec{b}|$.

11. Find the distance between the point $A = (-7, 4)$ and $B = (-5, -1)$.

12. Find the equation of the line whose y-intercept is -2 and slope is $\frac{3}{2}$.

SECTION – B

II. Answer any **SIX** of the following. Each question carries **5** marks : **(6 × 5 = 30)**

13. If $A = \{1, 4\}$, $B = \{2, 3, 6\}$, $C = \{2, 3, 7\}$ then verify that

$A \times (B - C) = (A \times B) - (A \times C)$.

14. Show that the function $f : Q \to Q$ defined by $f(x) = 2x + 3$ is both one-one and onto. Here Q is the set of all rational numbers.

15. Prove that $p \vee (q \wedge r) \leftrightarrow [(p \vee q) \wedge (p \vee r)]$ is a Tautology.

16. Write the converse, inverse and contra-positive of the conditional. "If two integers are equal then their squares are equal".

17. Prove that $(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$.

18. Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

19. Using Cramer's rule solve :

$5x - y - 4z = 5$, $2x + 3y + 5z = 2$, $7x - 2y + 6z = 5$.

20. Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

SECTION – C

III. Answer any **SIX** of the following. Each question carries **5** marks : **(6 × 5 = 30)**

21. If $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$, show that $a^2 + b^2 = 7ab$.

22. How many three digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

23. If $2nc_3 : nc_3 = 11:1$ find n.

24. Prove that the set $G = \{1, -1, i, -i\}$ form an abelian group under multiplication.

25. Show that $H = \{0, 2, 4\}$ is subgroup of the group $(G, +_6)$, where $G = \{0, 1, 2, 3, 4, 5\}$

26. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$, verify that

    $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

27. Find the area of the triangle whose vertices are $A(3, 2, 1)$, $B(4, -1, 2)$ and $C(-1, 3, 2)$ using vector method.

28. Find the value of 'm', if $\vec{a} = m\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + \hat{k}$ are co-planar.

SECTION – D

IV. Answer any **FOUR** of the following. Each question carries **5** marks : **(4 × 5 = 20)**

29. Prove that the points (6, 4), (7, -2), (5, 1) and (4, 7) form the vertices of a parallelogram.

30. Find the ratio in which the points $p(2, 7)$ divides the line joining the points $A(8, 9)$ and $B(-7, 4)$.

31. Find the equation of the perpendicular bisector of the line joining the points $A(3, -2)$ and $B(4, 1)$.

32. Find the value of k such that the line $(k-2)x + (k+3)y - 5 = 0$ is perpendicular to the line $2x - y + 7 = 0$.

33. If the acute angle between the lines $4x - y + 7 = 0$ and $k_x - 5y - 9 = 0$ is $45°$. Find the value of k.

34. Find the equation of the line passing through the point of intersection of the lines $2x + 3y - 7 = 0$ and $5x - 6y + 8 = 0$ and the point (4, 3).
